Feb 7

Logistic regression: adding predictors

1. Adding predictors to logistic regressions
2. Odds (and odds ratios) versus probabilities
3. Transforming posterior distributions
4. Prior predictive simulation in R

Cocaine use among adolescents

$$C_i \sim \text{Bernoulli}(p_i)$$

$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$

$$\beta \sim \boxed{\text{Norm}(0, 0.3)}$$

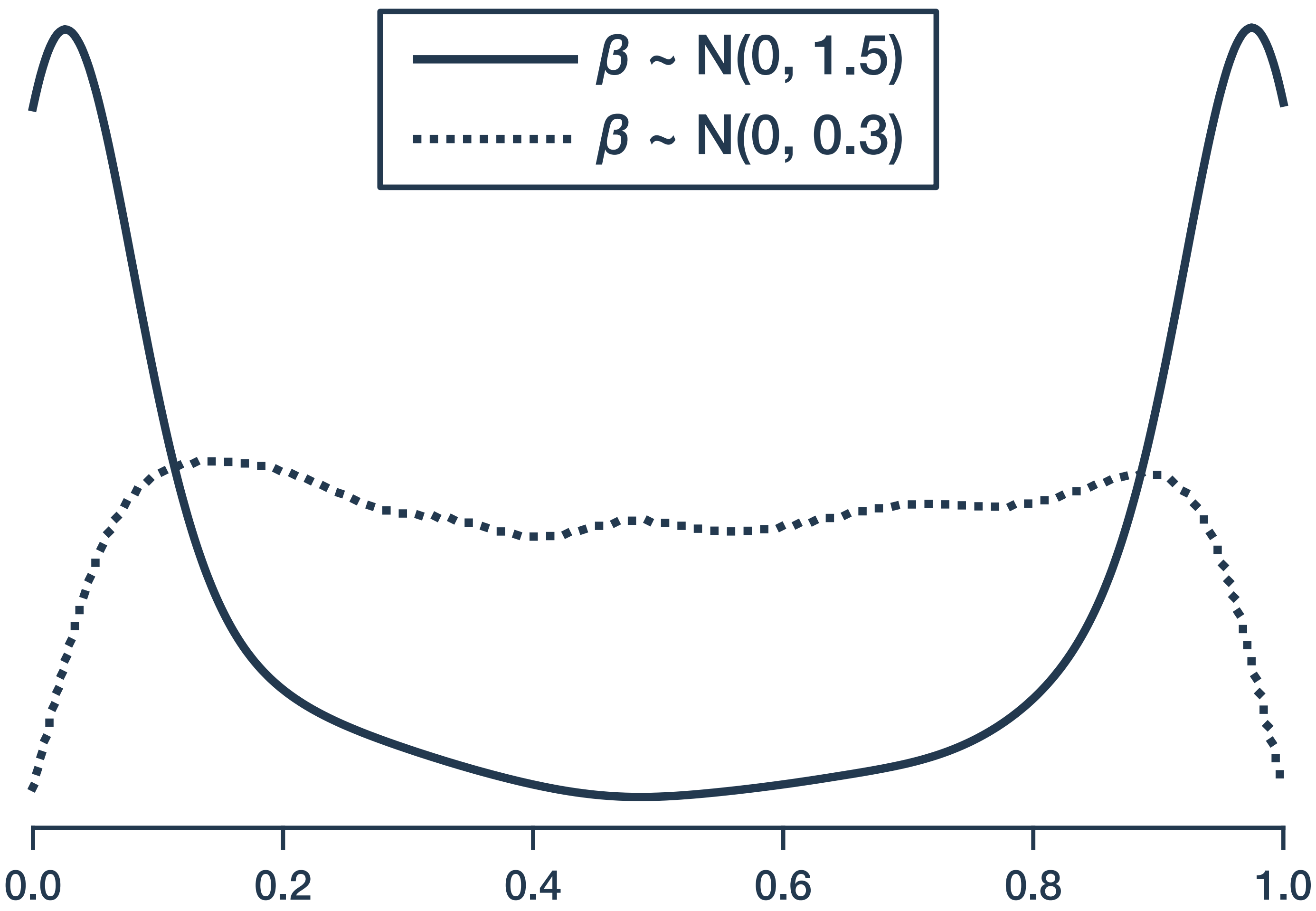
← **Where did this prior come from?**

Priors in logistic regressions

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$

Prior predictive simulation
(at minimum grade)



Logistic regression coefficients

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$
$$\beta \sim \text{Norm}(0, 0.3)$$

	Mean	90% HPDI	
α	-4.59	-5.41	-3.91
β	0.12	0.05	0.20

Interpreting α | "The expected probability of having tried cocaine for a student with G_i=0 is: **logistic(-4.59) = 0.01 = 1%**"

Standardized G_i | "The expected probability of having tried cocaine for a student in grade 9.54: **logistic(-4.59) = 0.01 = 1%**"

Logistic regression coefficients

Interpreting β

Odds ratio

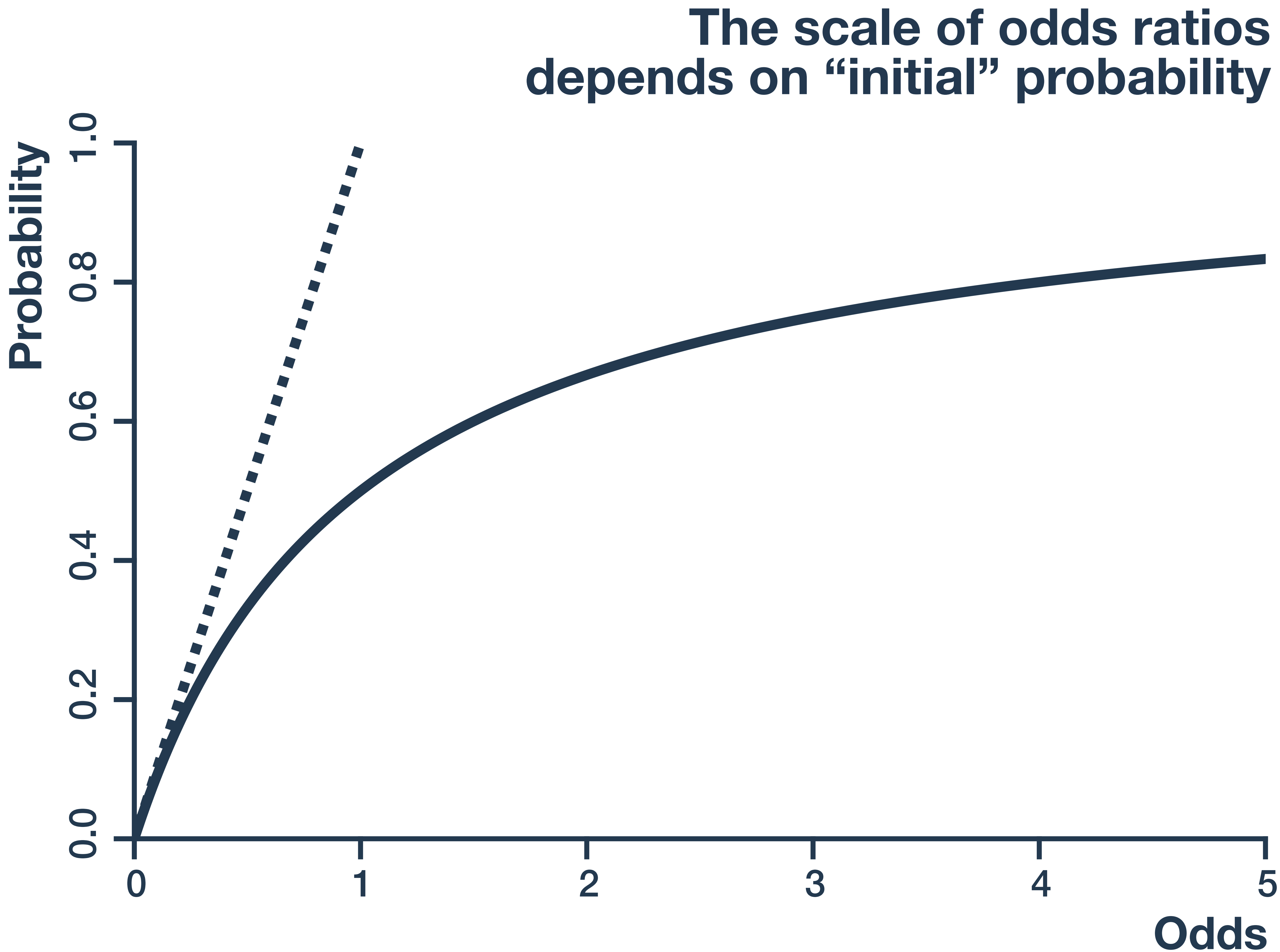
$$\exp(\beta) = \frac{\left(\frac{p^{G=1}}{1-p^{G=1}}\right)}{\left(\frac{p^{G=0}}{1-p^{G=0}}\right)}$$

$$\begin{aligned}\text{logit}(p) &= \alpha + \beta G \\ \log\left(\frac{p}{1-p}\right) &= \alpha + \beta G \\ \frac{p}{1-p} &= \exp(\alpha + \beta G) \\ \frac{p}{1-p} &= \exp(\alpha) \times \exp(\beta G)\end{aligned}$$

"For every unit increase in the covariate, the expected *odds* of the outcome is multiplied by exp(β)"

"For every 1.67 grades a student completes, their expected *odds* of trying cocaine increased by 13% (multiplied by 1.13)"

Odds ratios


The scale of odds ratios
depends on “initial” probability
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
Odds

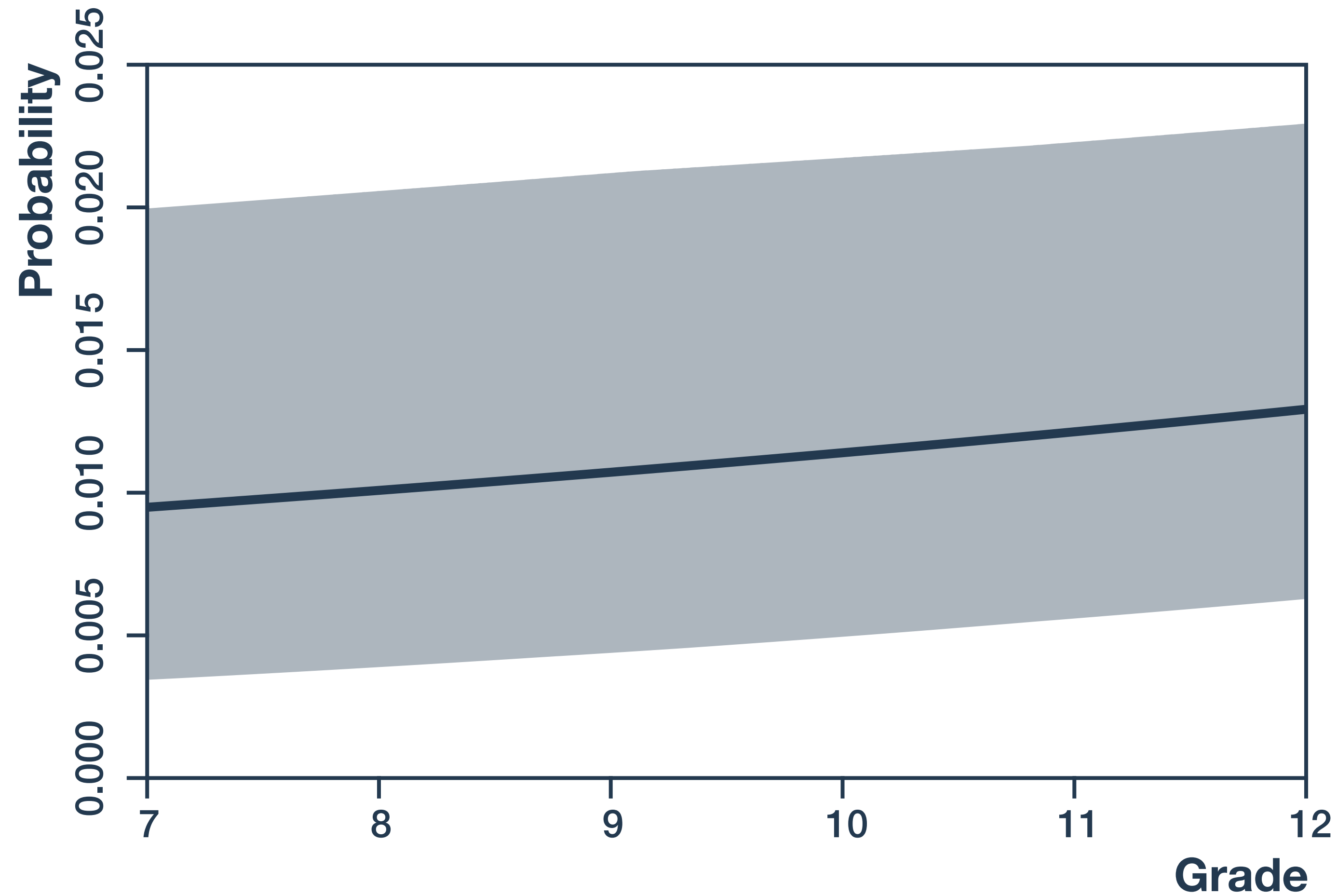
Logistic regression coefficients

Alternatives to odds ratios

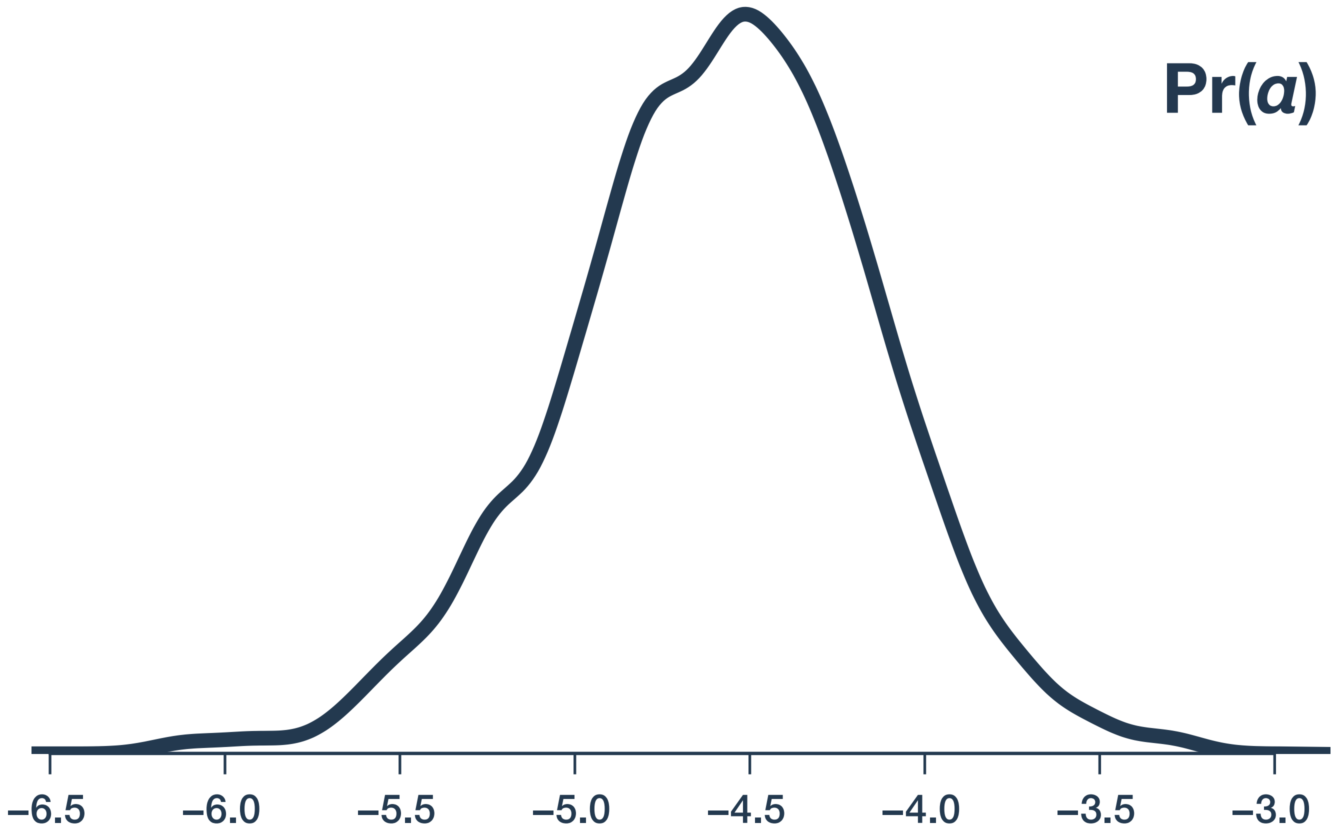
Cases

“ An average 7th grade student has about a 0.83% chance of having tried cocaine, while for an average 12th grader, that probability is about 1.21%”

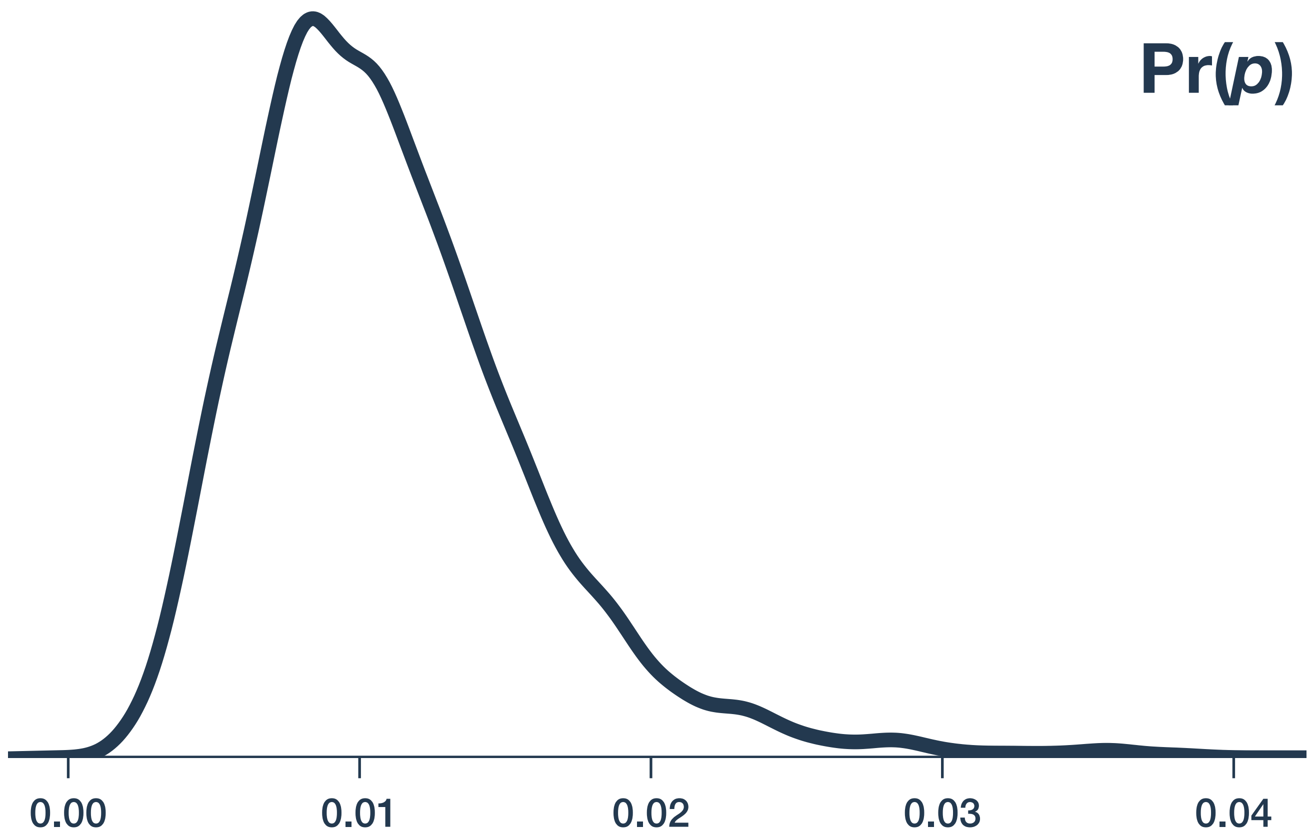
Posterior visualization



Logistic posteriors



Symmetric posterior distribution for $\alpha + \beta G_i$



Skewed posterior distribution for $p = \text{logit}^{-1}(\alpha + \beta G_i)$

Adding covariates

G_i: Grade in school (standardized)

D_i: Delinquency (standardized)

W_i: White (indicator)

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = a + \beta_G G_i + \beta_D D_i + \beta_W W_i$$

$$a \sim \text{Norm}(0, 1.5)$$
$$\beta_G \sim \text{Norm}(0, 0.5)$$
$$\beta_D \sim \text{Norm}(0, 0.5)$$
$$\beta_W \sim \text{Norm}(0, 0.5)$$

	Mean	90% HPDI	
a	-6.26	-7.25	-5.30
β_G	0.18	0.09	0.27
β_D	0.90	0.77	0.99
β_W	0.70	0.40	1.00

Adding covariates

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta_G G_i + \beta_D D_i + \beta_W W_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$
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Interpreting results using *odds ratios*

Each coefficient has an easy-to-calculate (but hard-to-interpret) meaning

$\exp(\alpha) = 0.0019$ The odds of cocaine use for a student in the mean grade, with mean deviance, and who is not white is about 0.0019.

$\exp(\beta_G) = 1.2022$ For each standard deviation increase in grade level, students' odds of having tried cocaine increase by about 20%.

$\exp(\beta_D) = 2.4611$ For each standard deviation increase in deviance score, students' odds of having tried cocaine increase by about 146%.

$\exp(\beta_W) = 2.0164$ White students' odds of having dried cocaine are about 102% higher than non-white students'.

Adding covariates

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$$\alpha \sim \text{Norm}(0, 1.5)$$
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Interpreting results using *selected cases*

Don't interpret coefficients, but meaningful hypothetical cases in the data

$\text{logit}^{-1}(\alpha)$ = 0.0019 — The probability of cocaine use for a student in the mean grade, with mean deviance, and who is not white is about 0.0019.

$\text{logit}^{-1}(\alpha + 2\times\beta_G)$ = 0.0027 — In comparison, an otherwise-identical student whose grade level is two standard deviations above the mean has a probability of cocaine use of about 0.0027

$\text{logit}^{-1}(\alpha + \beta_W)$ = 0.0038 — An 'average' white students' probability of having dried cocaine is about 0.0038